CMPUT 626 - A2
Machine Learning and Practical Privacy

Thinking About Cryptography 1

Fall 2023, Tuesday/Thursday 3:30-4:50pm

Lecture Section Update

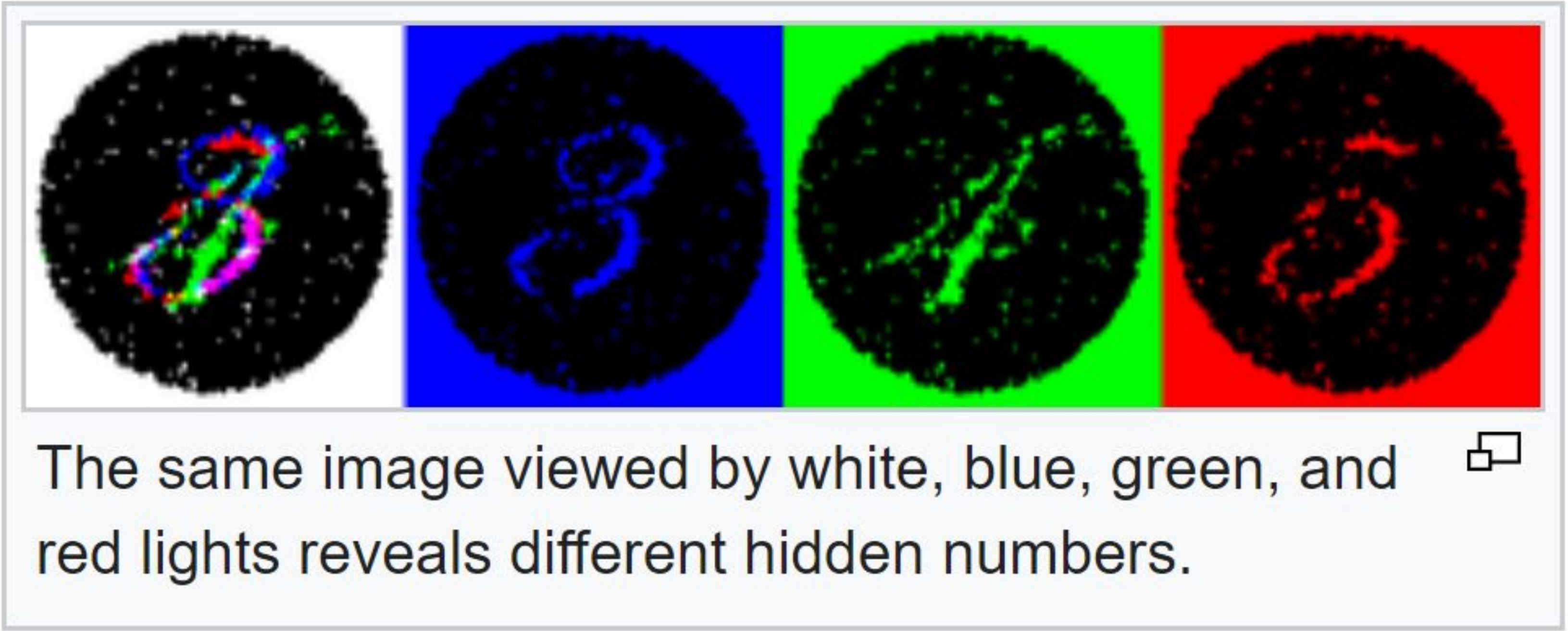
Week	Tuesday	Thursay
One: September 5th and 7th	Course overview, Privacy Part 1 Bailey	Cryptography Part 1 Bailey
Two: September 12th and 14th	Cryptography Part 2 Bailey	Ethics, law, and policy Bailey
Three: September 19th and 21st	Privacy Part 2 Bailey	Privacy Part 3 Bailey

Learning Outcomes

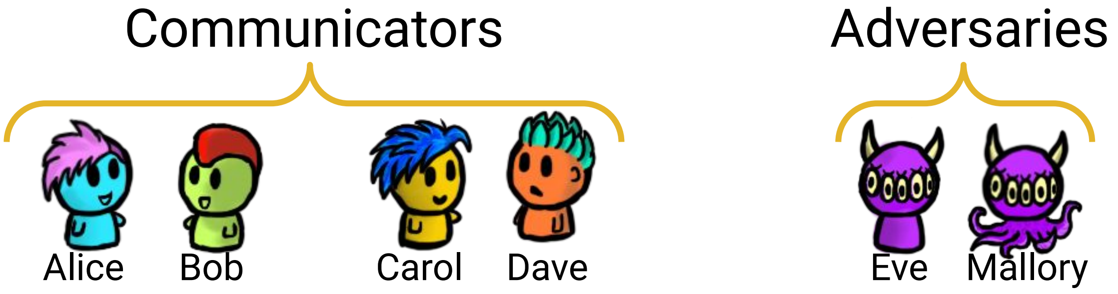
- Identify attack techniques and apply them (cryptanalysis)
- Explain building blocks of modern cryptography
- Explain how modern cryptography properties arose

Goal: Basically, know what cryptography tools exist and how to securely use them. <u>Build a foundation of primitives</u> for more complicated “applied cryptography” later.

Steganography- Secretly “hidden” messages

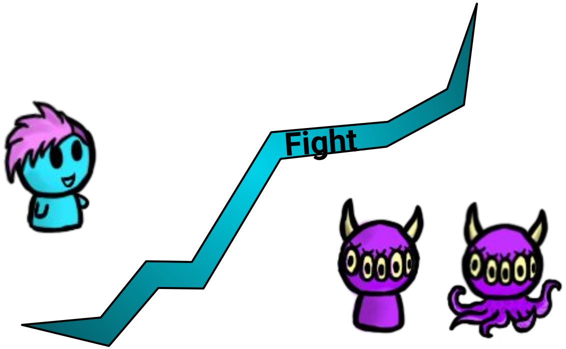


Cryptography - Writing “secret” messages

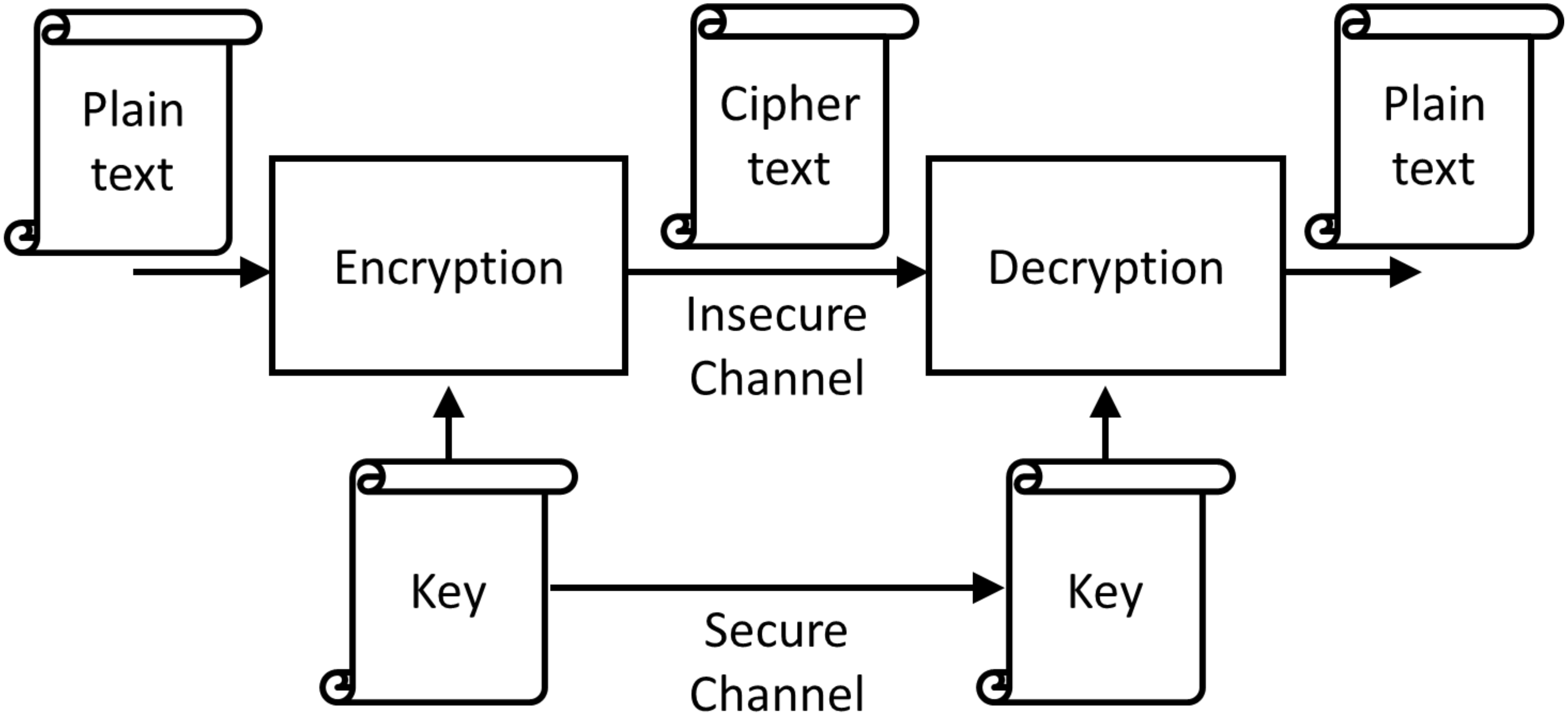


CIA and Cryptography

- Confidentiality, prevent Eve **reading** Alice’s messages
- Integrity, prevent Mallory from **changing** Alice’s messages
- Authenticity, Prevent Mallory from **impersonating** Alice



Cryptography - Path for Secret Messages



Historical Ciphers: Example One

FUBSWRJUDSKB

CRYPTOGRAPHY

Caesar Cipher

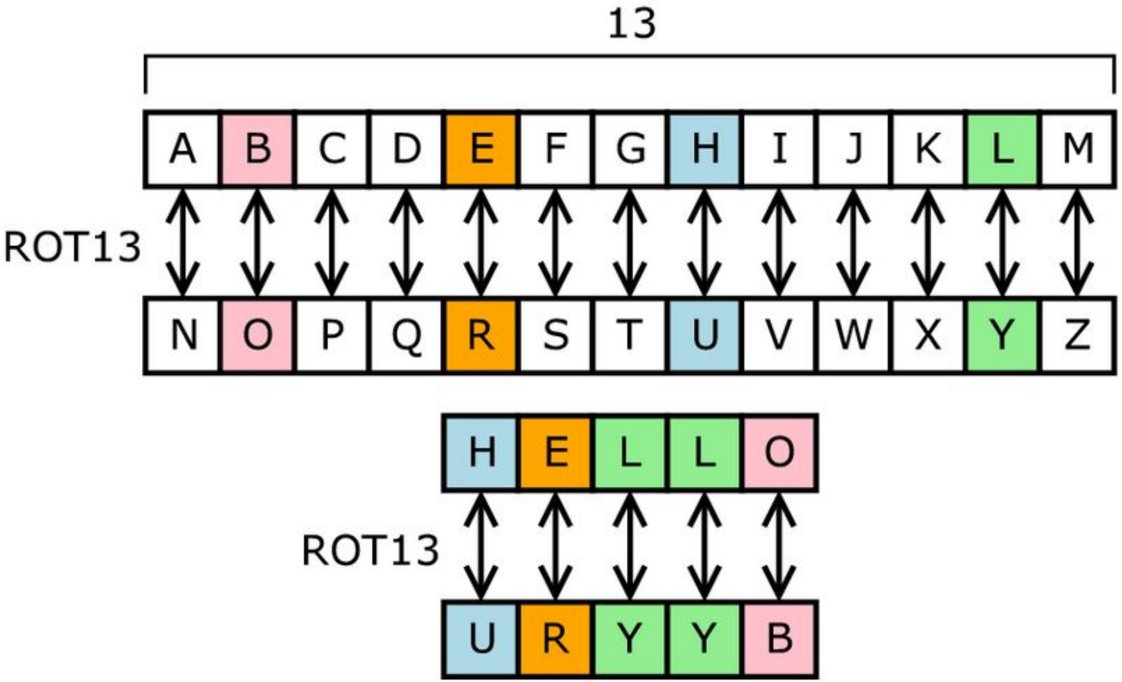


Image source: wikipedia

Cryptanalysis - Analyzing “secret” messages

Mwahaha

We will learn the secretsssss.

Historical Ciphers: Example Two

gsrh xlfihv rh zylfg xibkgltizksb uli gsv urihg gsivv dvvph. zmw gsvm zkkorvw xibkgltizksb uli kirezxb zmw hvxfirgb lu wzgz.

English Frequency

A	11.7%	
B	4.4%	
C	5.2%	
D	3.2%	
E	2.8%	
F	4%	
G	1.6%	
H	4.2%	
I	7.3%	
J	0.51%	
K	0.86%	
L	2.4%	
M	3.8%	

N	2.3%	
O	7.6%	
P	4.3%	
Q	0.22%	
R	2.8%	
S	6.7%	
T	16%	
U	1.2%	
V	0.82%	
W	5.5%	
X	0.045%	
Y	0.76%	
Z	0.045%	

Historical Ciphers: Example Two

gsrh xlfihv rh zylfg xibkgltizksb uli **gs**v urihg **gs**ivv dvvph. zmw **gs**vm zkkorvw xibkgltizksb uli kirezxb zmw hvxfirgb lu wzgz.

Historical Ciphers: Example Two

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This course is about cryptography for **th**e first **th**ree weeks. And **th**en applied cryptography for privacy and security of data.

Kerckhoff Principle

The security of a cryptosystem should solely depend on the secrecy of the key, but never on the secrecy of the algorithms.

Historical Ciphers: Example Three

LECTURE SECURITY AND CRYPTOGRAPHY I

↓

LENGECDRCUCATRRPUIYHRTPYEYTISAO

Historical Ciphers: Example Three

LECTURES

ECURITYA

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LENGECDRCUCATRRPUIYHRTPYEYTISAO

Historical Ciphers: Example Three

Shannon's maxim!!!! (design assuming they'll learn the algorithm

LENGECDRCUCATRRPUIYHRTPYEYTISAO

Shannon’s Maxim and Kerkhoff’s Principle Mean:

- Security shouldn’t rely on the secrecy of the method
- Do use <u>public</u> algorithms with <u>secret</u> “keys”
- The adversaries target…is the key

Key: Easier to change a “short” key than your whole system.
(e.g., Recovery)

Unconditionally Secure: One-Time Pad

Message: $x_0 \; x_1 \; x_2 \; \dots \; x_n$

$\oplus$

Key: $k_0 \; k_1 \; k_2 \; \dots \; k_n$

$=$

Ciphertext: $y_0 \; y_1 \; y_2 \; \dots \; y_n$

Rule: $y_i = x_i + k_i \pmod{2}$

Provably Security for One-Time Pad

<Ciphertext is uniformly distributed independent of the plaintext distribution>

$x_i = 0$ with probability p ($x_i = 1$: 1-p),

$k_i = 0$ with probability 0.5 ($k_i = 1$: 0.5), $y_i = 0$ with probability:

$$
\begin{aligned}
p(y_i = 0) &= p(x_i = 0)\, p(k_i = 0) + p(x_i = 1)\, p(k_i = 1) \\
&= 0.5p + 0.5(1-p) \\
&= 0.5
\end{aligned}
$$

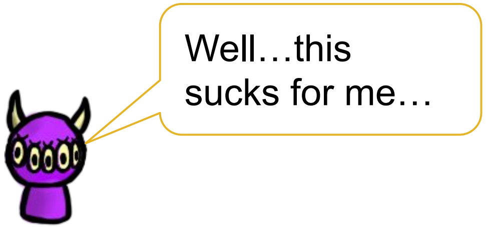
Provably Secure Con’t

Every ciphertext y can be decrypted **into every arbitrary plaintext** x using the key

$$\mathbf{k = yx}$$

Consequently the ciphertext cannot contain any information about the plaintext

Encryption is “deniable”



Act.

What if it is a many-time pad?

Key: K

$\text{Ciphertext}_1 = \text{message}_1$ xor K = 2c1549100043130b1000290a1b

$\text{Ciphertext}_2 = \text{message}_2$ xor K = 3f16421617175203114c020b1c

Your turn, goal: Learn the ciphertexts.

Many-time pad? Messages Lack True Randomness

C_1

C_2

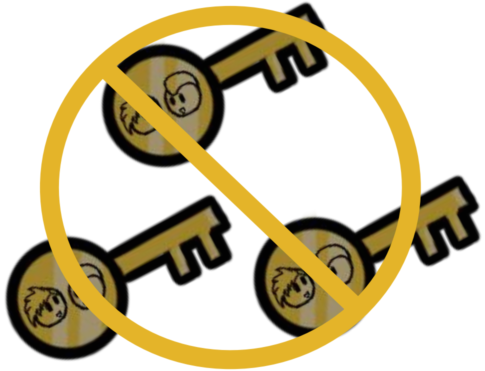
$C_1 \oplus C_2$

M_2

M_1

One-Time Pad - Conditions...

- Key as long as the message
- Key uniformly random
- Only used once



So…Cryptography?

- Simple substitution/transposition is computationally insecure
- One-Time Pad is inefficient over the secure channel

Goal: Securely communicate “a lot” of information on an insecure channel while requiring “limited” communication over a secure channel

Recap: A, B, C versus A and B and C

Substitution is insecure…

Transposition is insecure…

Key reuse using XOR (one-time pad) is insecure…

BUT

Repeat it often enough and it can be widely regarded as secure

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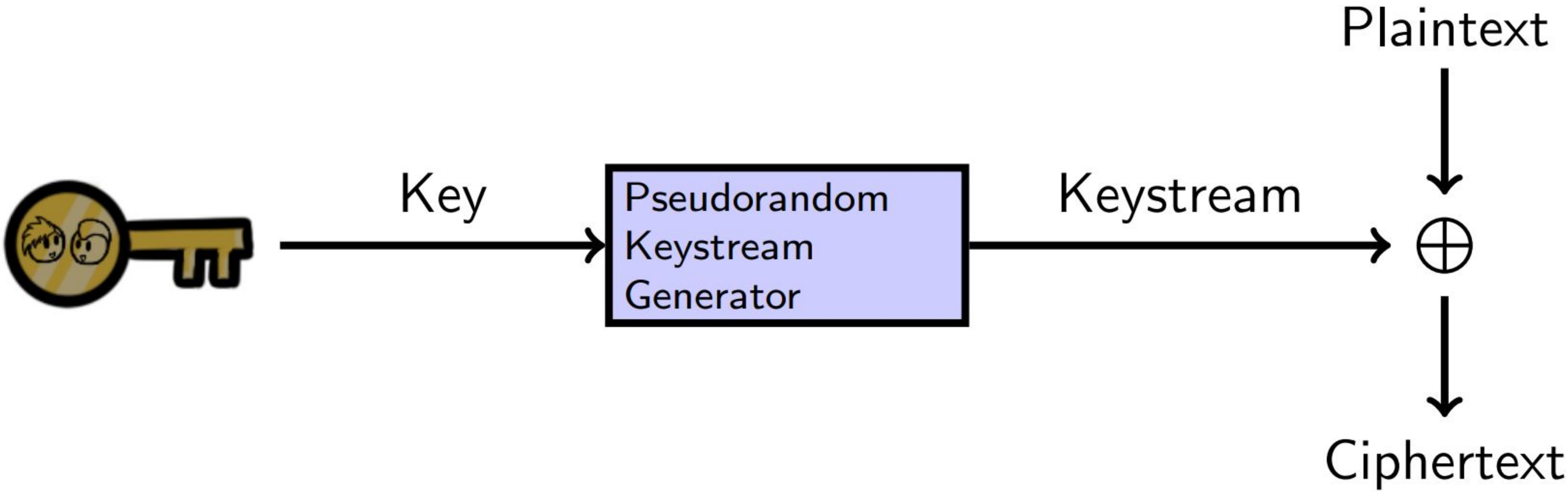
Stream Ciphers and Block Ciphers

Stream Cipher?

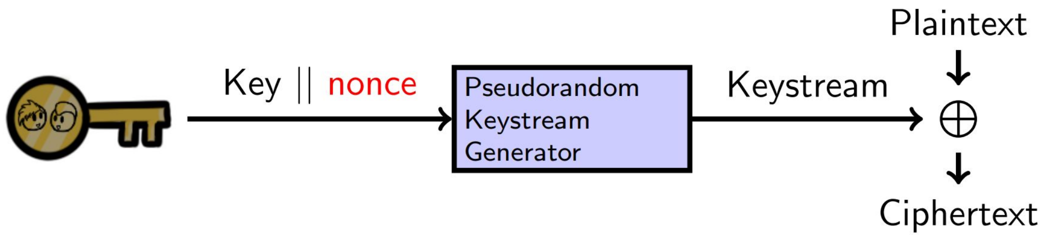


Fun(?) Facts:

- RC4 was the most common stream cipher on the Internet but deprecated.
- ChaCha increasingly popular (Chrome and Android), and SNOW3G in mobile phone networks.

Stream Ciphers Share Conditions with OTP

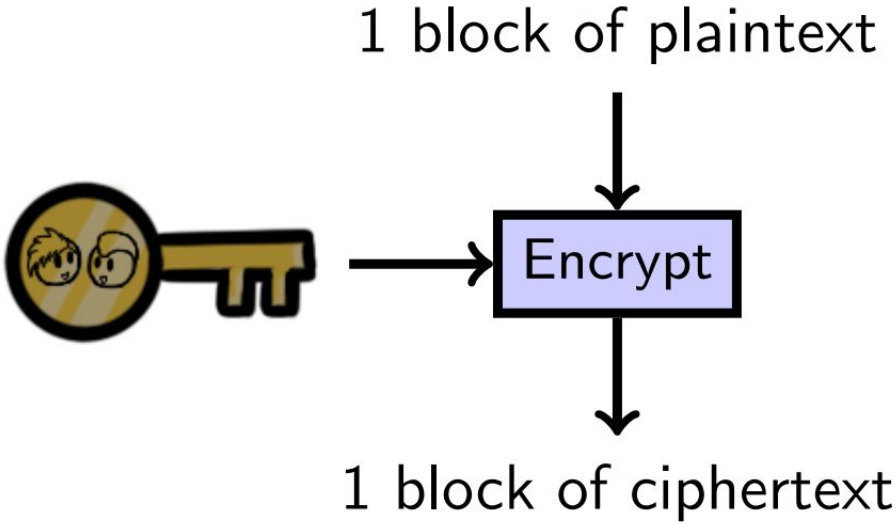
- Stream ciphers can be very fast
  - This is useful if you need to send a lot of data securely
- But they can be tricky to use correctly!
  - We saw the issues of re-using a key! (two-time pad)
  - Solution: concatenate key with nonce (we'll see more about nonces later)



Fun(?) Facts:

- WEP, PPTP are great examples of how not to use stream ciphers

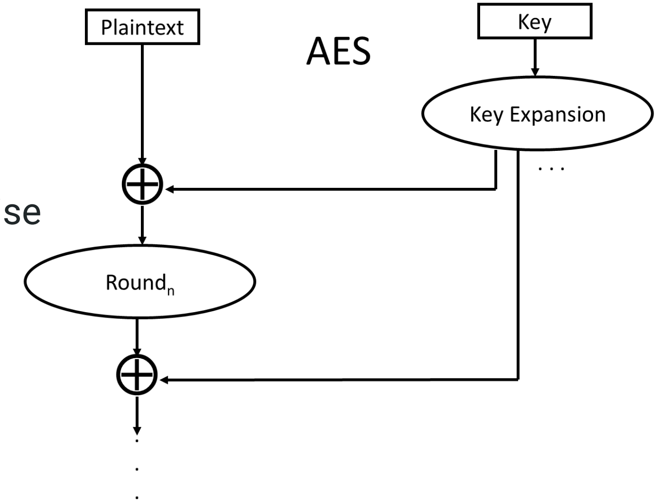
Bit by bit....do you have to?



Block ciphers!!!

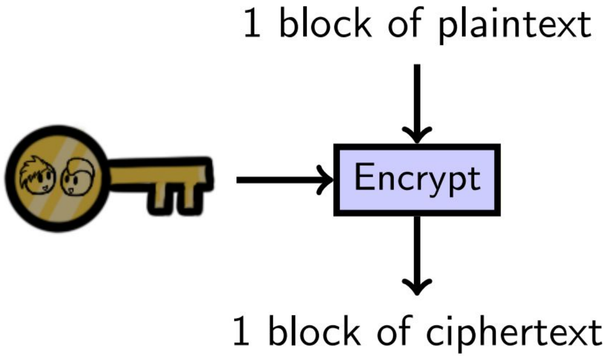
Block Ciphers

- Weakness of streams…one bit at a time?
  - What happens in a stream cipher if you change just one bit of the plaintext?
- Welcome, use of block ciphers
  - Block ciphers operate on the message one block at a time
  - Blocks are usually 64 or 128 bits long
- **AES**, the current standard
  - You better have a very…very good reason to choose otherwise

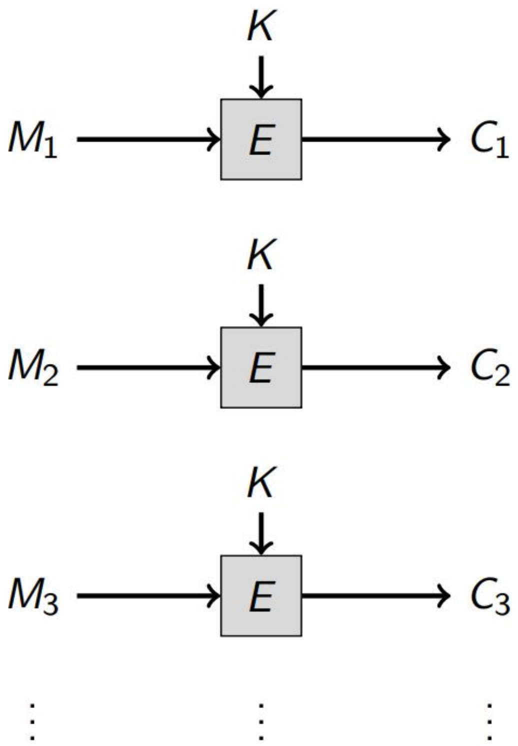


Two Catches with Block Ciphers

- Message is shorter than one block
  - padding
- Message is longer than a block
  - Modes of operation <new concept>

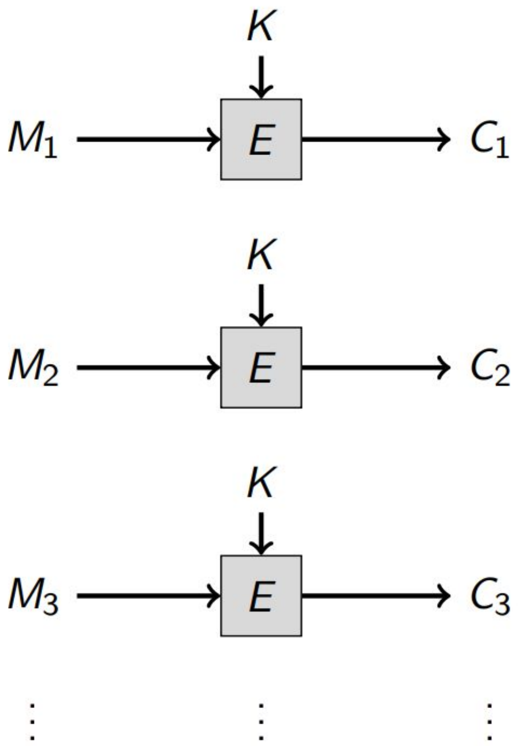


Block Ciphers and Modes of Operation: ECB Mode



- ECB: Electronic Code Book
- Encrypts each successive block separately

Block Ciphers and Modes of Operation: ECB Mode



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Q: What happens if the plaintext M has some blocks that are identical, $M_i = M_j$?

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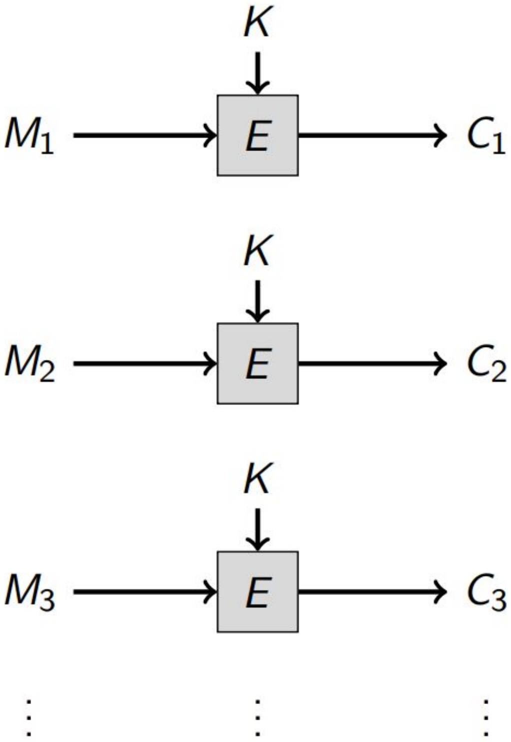

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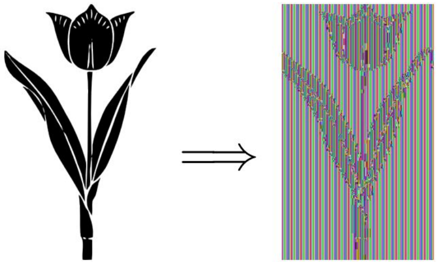
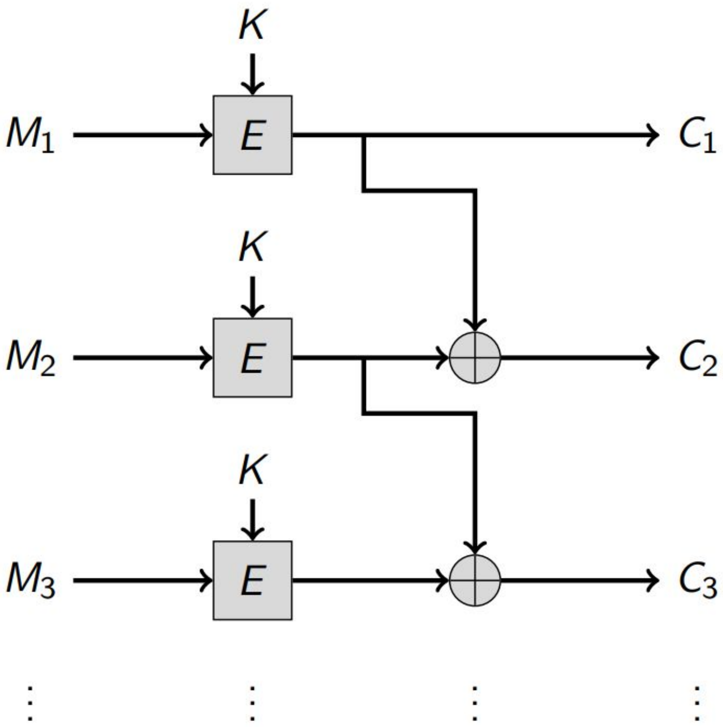
Q: What happens if the plaintext M has some blocks that are identical, $M_i = M_j$?

A: $C_i = E_K(M_i)$, $C_j = E_K(M_j) \Rightarrow C_i = C_j$



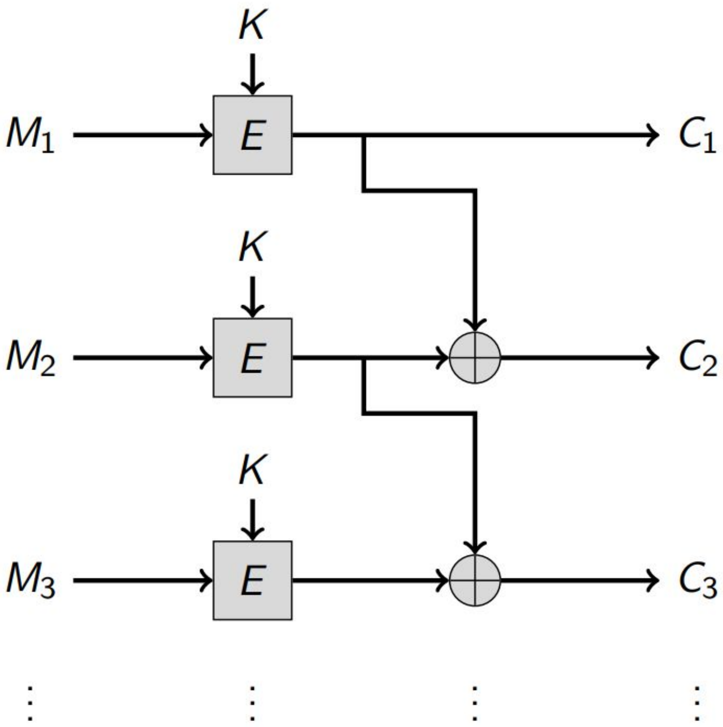
Attempt 1: Fixing ECB$_1$



- Provide “feedback” among different blocks, to avoid repeating patterns…

Q: Fix repeating patterns? Are there other issues?

Attempt 1: Fixing ECB[1]

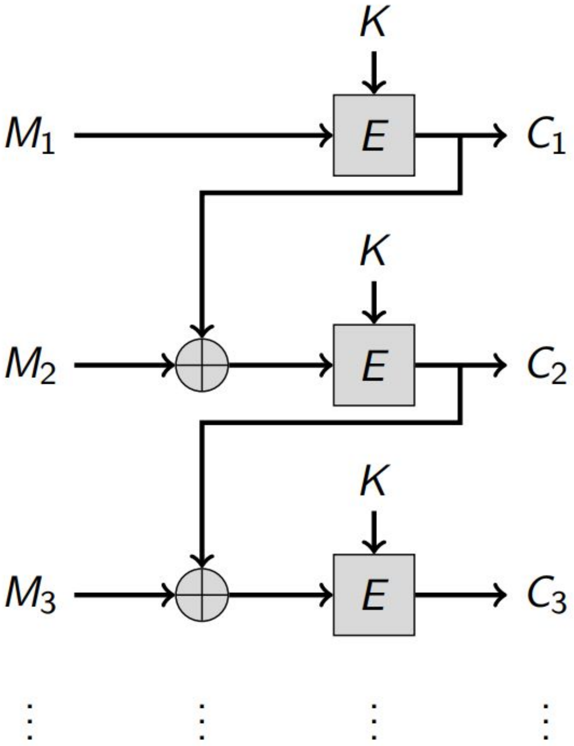


- Provide "feedback" among different blocks, to avoid repeating patterns…

Q: Fix repeating patterns? Are there other issues?

A: We can un-do the XOR <u>if we get all the ciphertexts</u>. This basically does not improve compared to ECB.

Attempt 2: ECB_2!!!

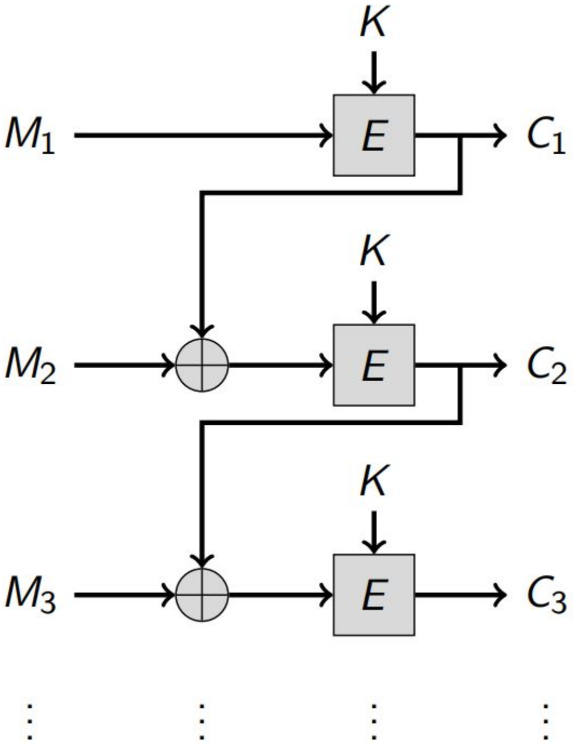


Q: Spot the difference?

Q: Is it fixed this time?

Q: Does this avoid repeating patterns among blocks?

Attempt 2: ECB_2!!!



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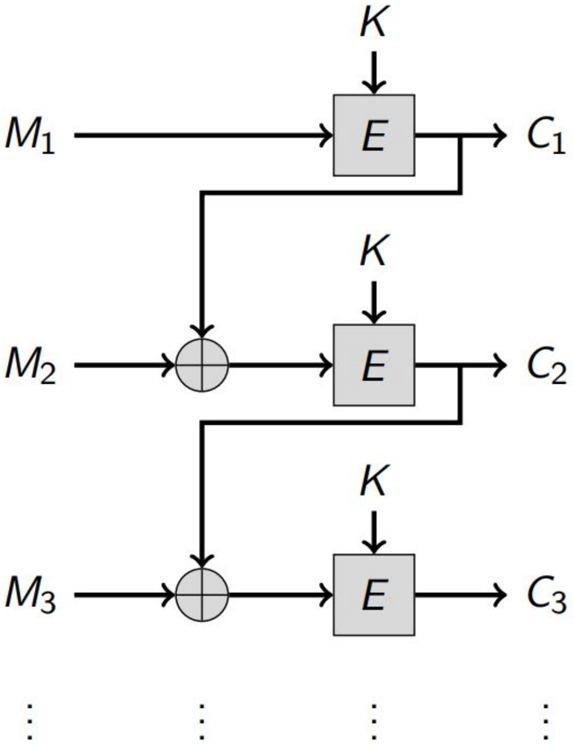

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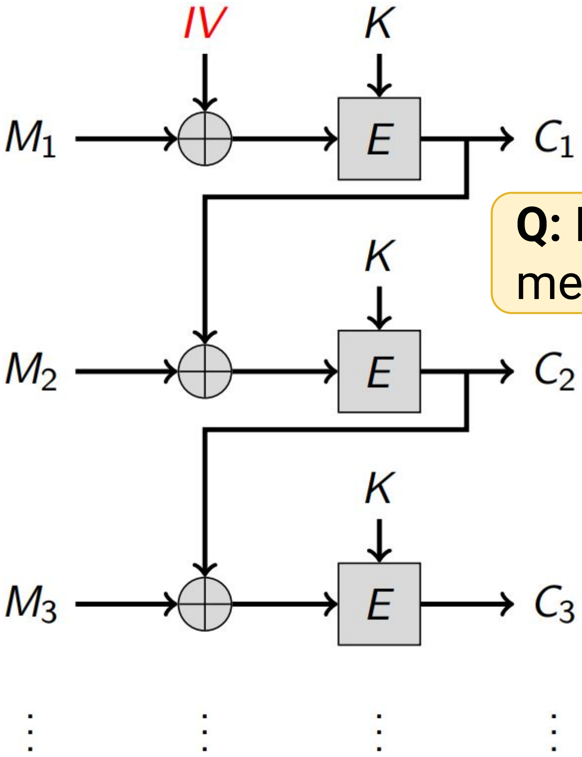
Q: What would happen if we encrypt the **message twice** with the **same key**?

A: $C_1 = E_K(M), C_2 = E_K(M) \Rightarrow C_1 = C_2$

New Plan: CBC Mode



Q: Does this solve the issue of encrypting equal blocks?

Q: Does this solve the issue of encrypting equal messages/plaintexts?

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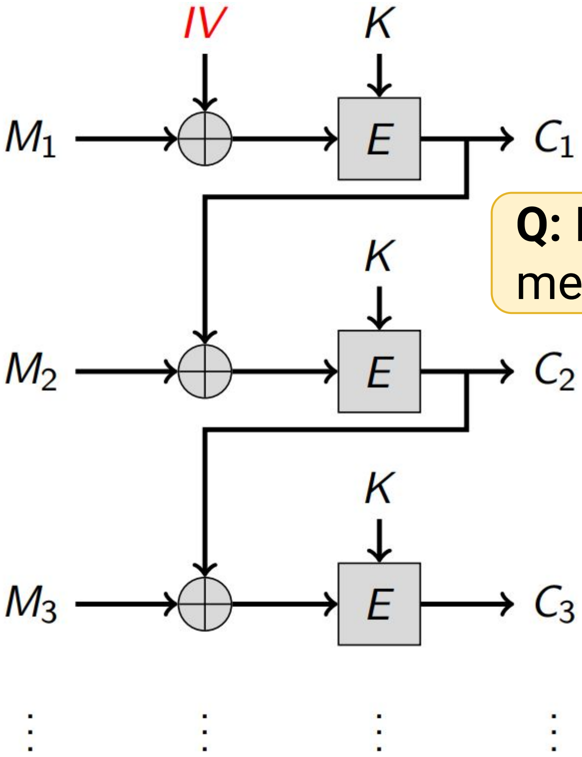

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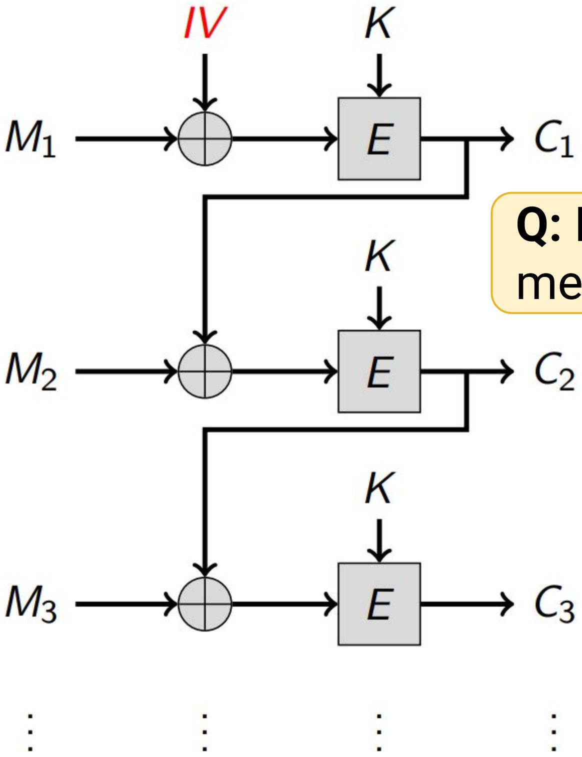

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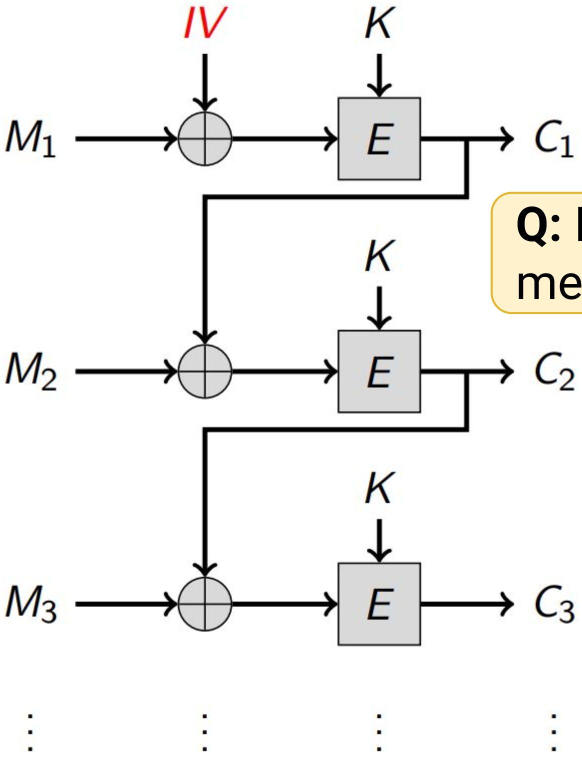
Q: Can we share IV in the clear?

New Plan: CBC Mode



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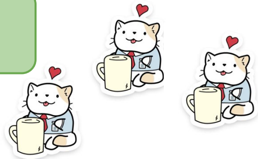
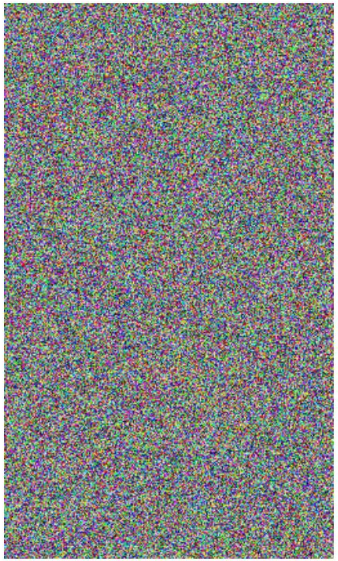


IV, an initialization vector, nonce, salt.

Modes of Operation Collection

- Cipher Block Chaining **(CBC)**, Counter **(CTR)**, and Galois Counter **(GCM)** modes
- Patterns in the plaintext are no longer exposed because these modes involve some kind of “feedback” among different blocks.
- But you need an **IV**



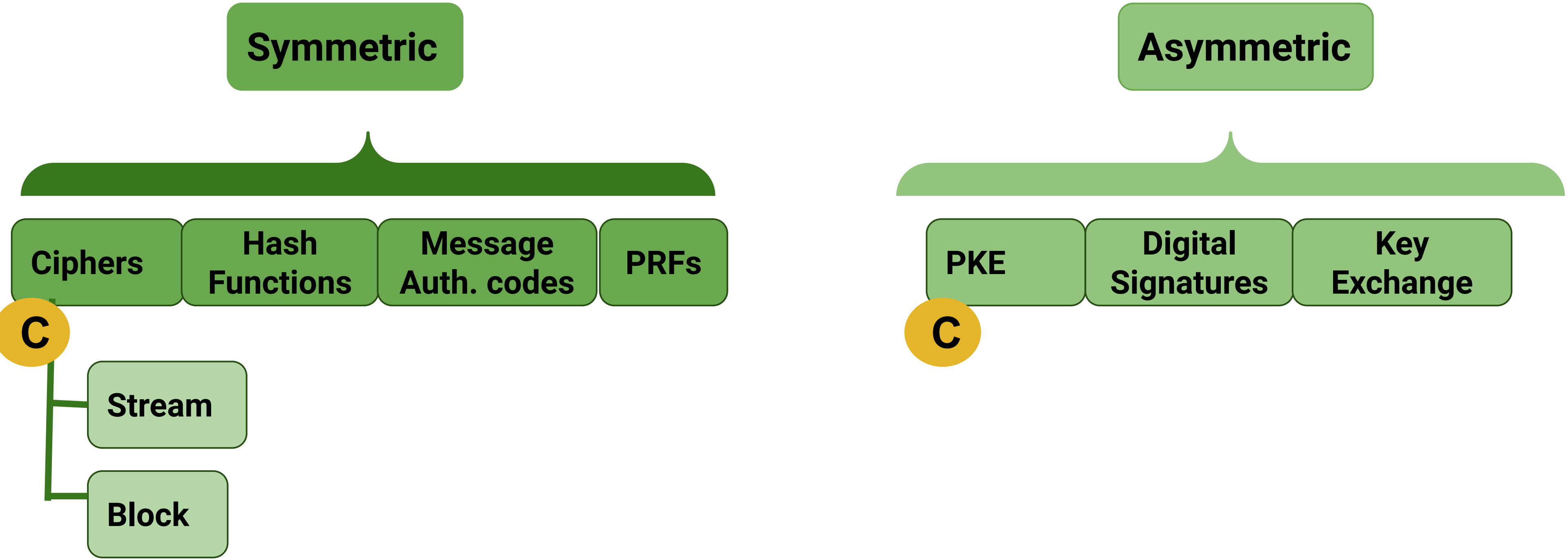
So...now what?

- How do Alice and Bob share the secret key?
    - Meet in person; diplomatic courier...
- In general this is very hard

Or, we invent new technology!!

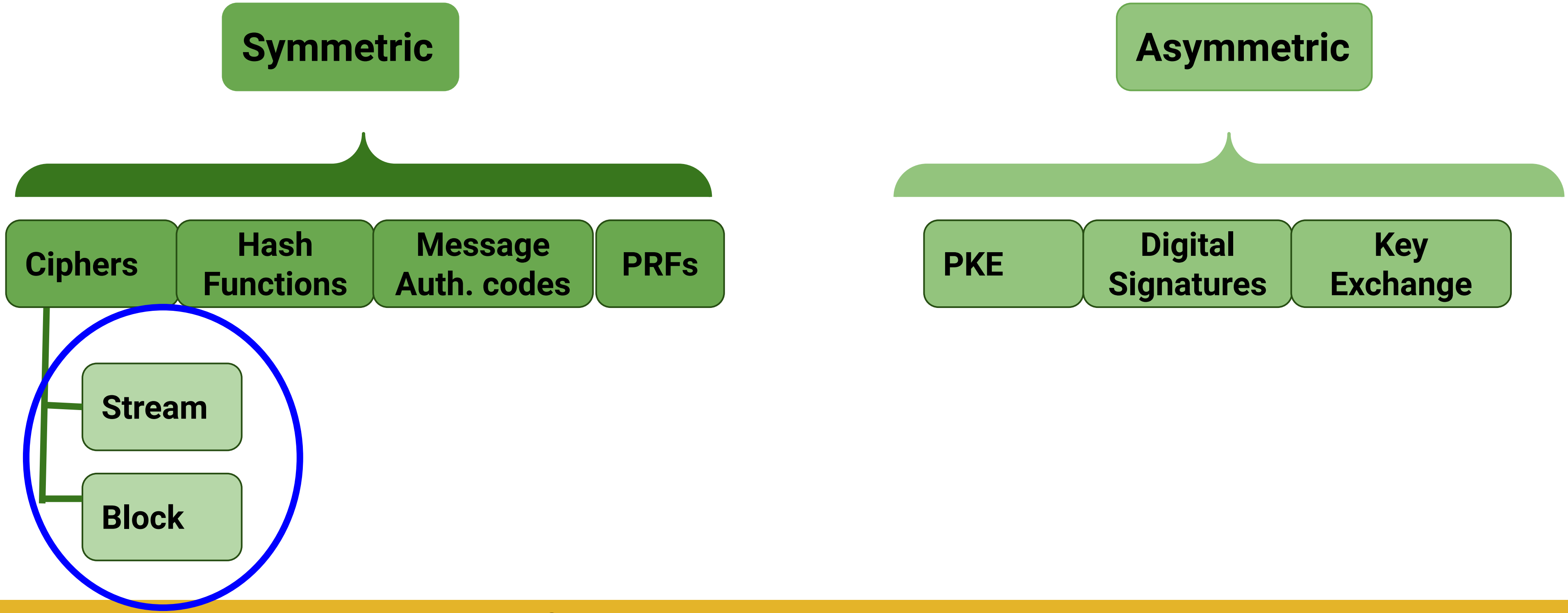
Spoiler Alert: it's already been invented...

Cryptography Organization



Organization display source: Doug Stebila

Cryptography Organization



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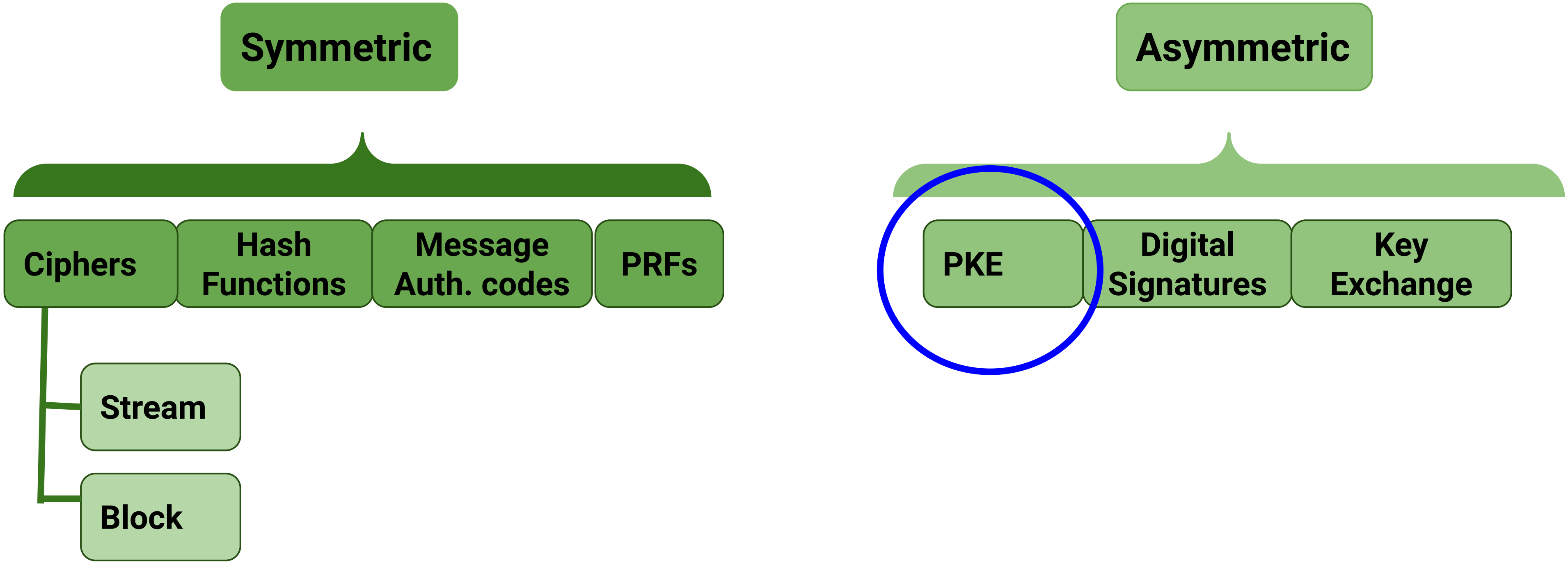
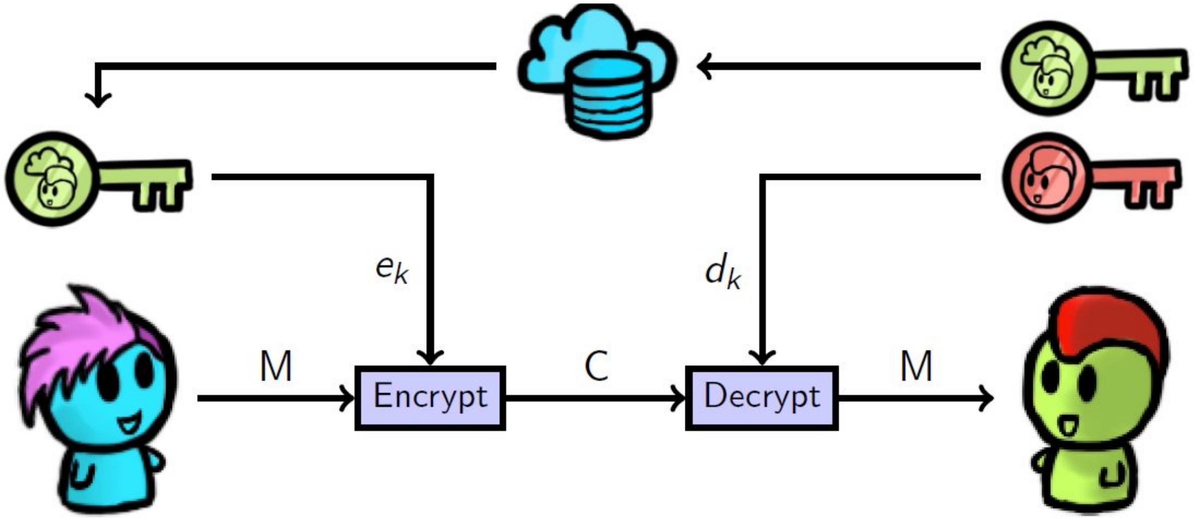


Public Key Cryptography, “1970s”



Examples:

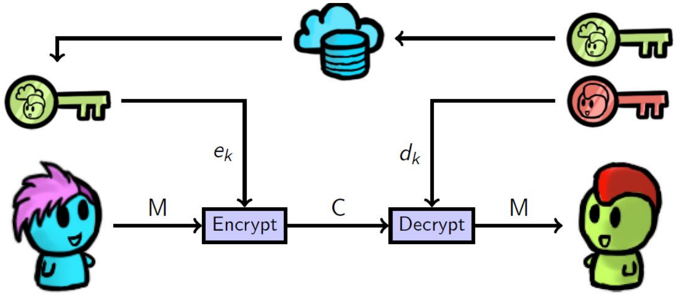
- RSA, ElGamal, ECC, NTRU

Steps for Public Key Cryptography?

1. Bob generates pair 

2. Bob gives everyone the public key 

3. Alice encrypts m and sends it



4. Bob decrypts using private key

5. Eve and Alice can’t decrypt, only have encryption key

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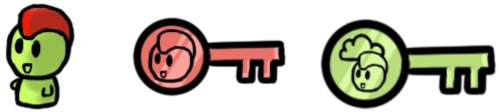
1. Bob generates pair 

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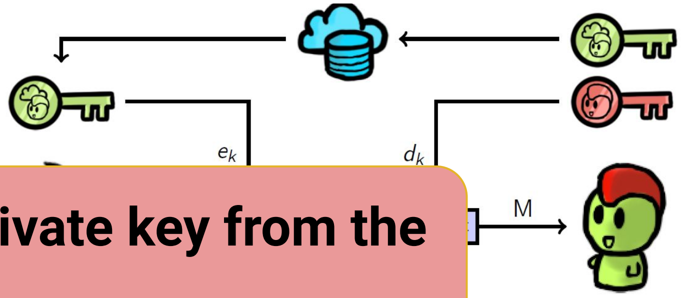

4. Bob d

It must be hard to derive the private key from the public key

5. Eve and Alice can't decrypt, only have encryption key

Requirements for PKE

- The encryption function? Must be easy to compute 
- The inverse, decryption? Must be hard for anyone without the key  vs. 

Thus, we require so called "one-way" functions for this.

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Thus, we require so called “one-way” functions for this.

Because of encryption, also injective

Because of decryption, we need a “trapdoor”

Time for Textbook RSA

- Computational difficulty of the **factoring problem**
  - Given two large primes n = p*q, it is very hard to factor n.
- Modular arithmetic: integer numbers that “wrap around”
- Overview:

$$(m^e)^d \equiv m \pmod{n}$$

Easy for me to pick e, d, and n that satisfy that equation

Ugh. I know e and n (even m) and can’t find d!!!

Fun (?) Facts::

- RSA first popular public-key encryption method, published in 1977

Textbook RSA (Simplified Overview)

1. Choose two **"large primes"** *p* and *q* (secretly)
2. Compute $n = p*q$
3. "Choose" value e and find d such that $(m^e)^d \equiv m \pmod{n}$
4. **Public key**: (e, n)
5. **Private key**: d (other numbers tossed)
6. Encryption: $c \equiv m^e \pmod{n}$
7. Decryption: $c^d \pmod{n}$

Textbook RSA (Simplified Overview)

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4. **Public key**: (e, n)
5. **Private key**: d (other numbers tossed)
6. Encrypt
7. Decrypt

Decryption works, but factoring n breaks this!

Note:

- RSA? Rivest, Shamir, and Adleman

A Closer Look at RSA: Rec… Para…eters

1. Choose two **"large primes"**
2. Compute n = p*q
3. "Choose" valu…
4. **Public ke…**
5. **Privat…**)
6. Encryptio…
7. Decryp…n: c^d (r… d n)

An "obvious" attack is for Eve
to attempt to factor *n*.
Then, compute c and e as
Bob would…

Note:

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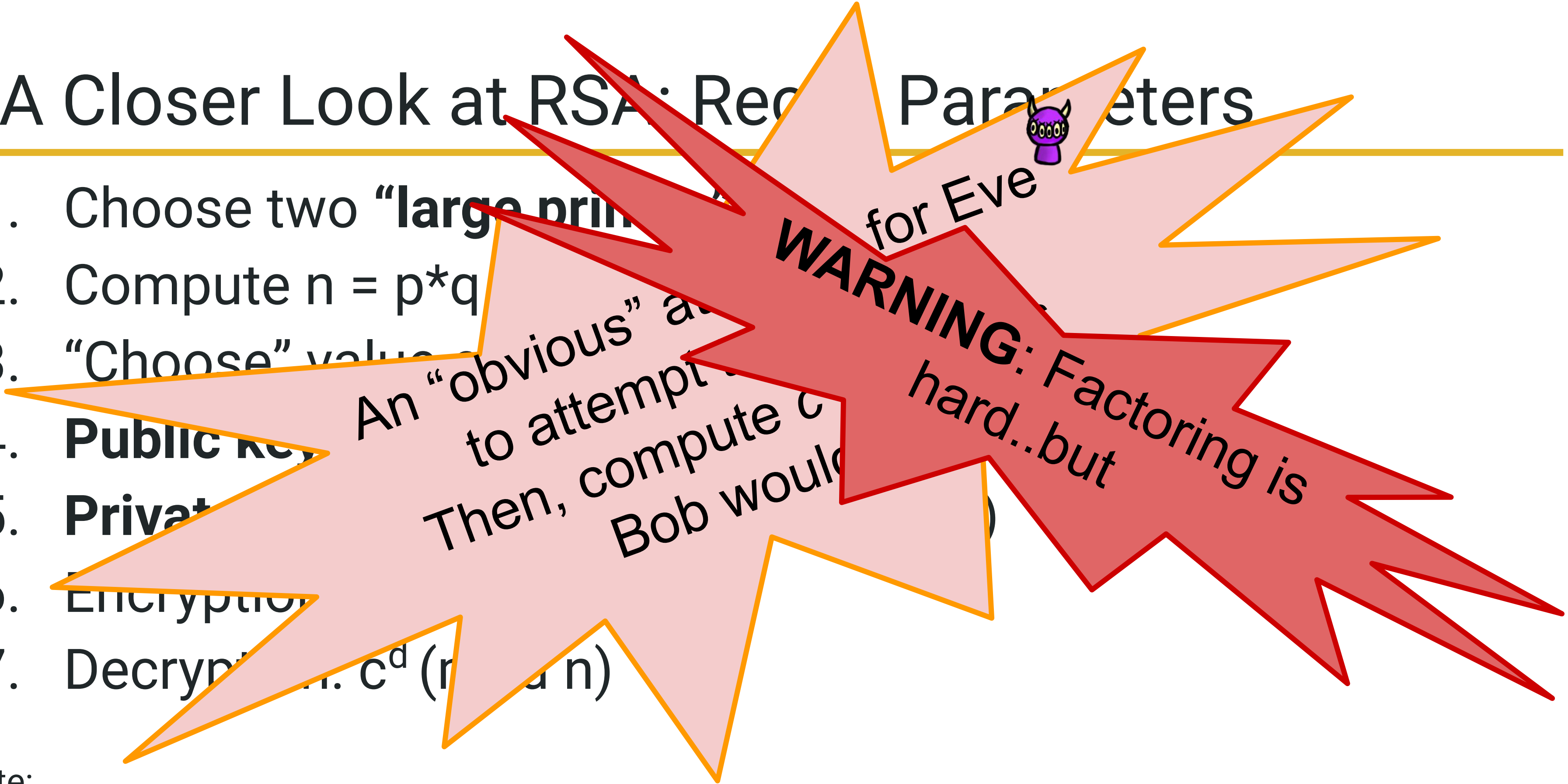


Note:

- RSA? Rivest, Shamir, and Adleman

Factoring and RSA

- You want to factor the public modulus? 
- Good news, abundant literature on factoring algorithms
- Bad news, “appropriate” primes will not be defeated

Bad primes: easily factored

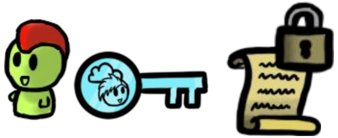
Malleability

$$\mathbf{A:}\ (m_1)^e * (m_2)^e = (m_1 * m_2)^e$$

It is possible to transform a ciphertext into another ciphertext that decrypts to a related plaintext

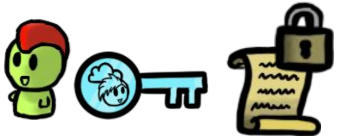
Undesirable (most of the time)

RSA and a Chosen Ciphertext Attack

- Alice is using RSA, public key (e, n) 
- Bob sends $c = E_e(m)$ 
- We are Eve! We snag c. 
- Alice…is confident about textbook RSA, will decrypt any ciphertext except c for us

Goal: Ask Alice to decrypt something (other than c) that helps us learn m

Executing CCA on Textbook RSA

- Alice is using RSA, public key (e, n) 
- Bob sends $c = E_e(m)$ 
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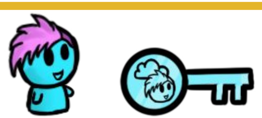
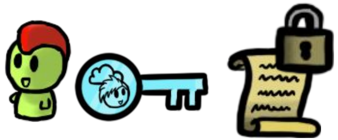
- Alice is using RSA, public key (e, n) 
- Bob sends $c = E_e(m)$ 
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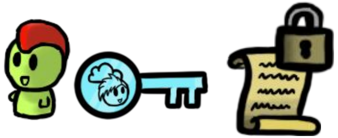
- Bob sends $c = E_e(m)$ 
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I am so clever mwahaha

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Textbook RSA: vulnerable to CCA
Note: Can be addressed with padding techniques

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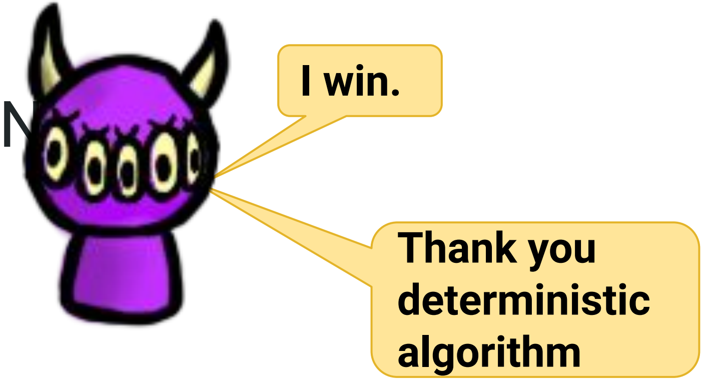

Adversaries and their Goals

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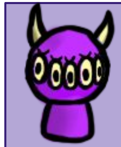
…but less ambitious goals can be very effective…

Adversaries and their Goals

Goal 1: Total Break

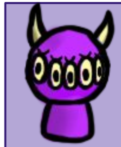


- Win the secret key k or
- Win Bob's private key k_b
- Can decrypt any c_i *for:*

$$c_i = E_k(m) \text{ or } c_i = E_{kb}(m)$$

- All messages using compromised k revealed
- Unless **detected** game over

Goal 2: Partial Break

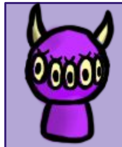


- Decrypt **a ciphertext** c (without the key)
- Learn **some** specific information about a message m from c

**Need to occur with non-negligible probability.

- **Some (or a)** message revealed

Goal 3: Distinguishable Ciphertexts



- *P{learn b* ∈ {0,1}} exceeds ½
- Distinguish between $E(m_1)$ and $E(m_2)$ or between E(m) and E(random string)

- The ciphertexts are leaking small/some information…

Until next time...